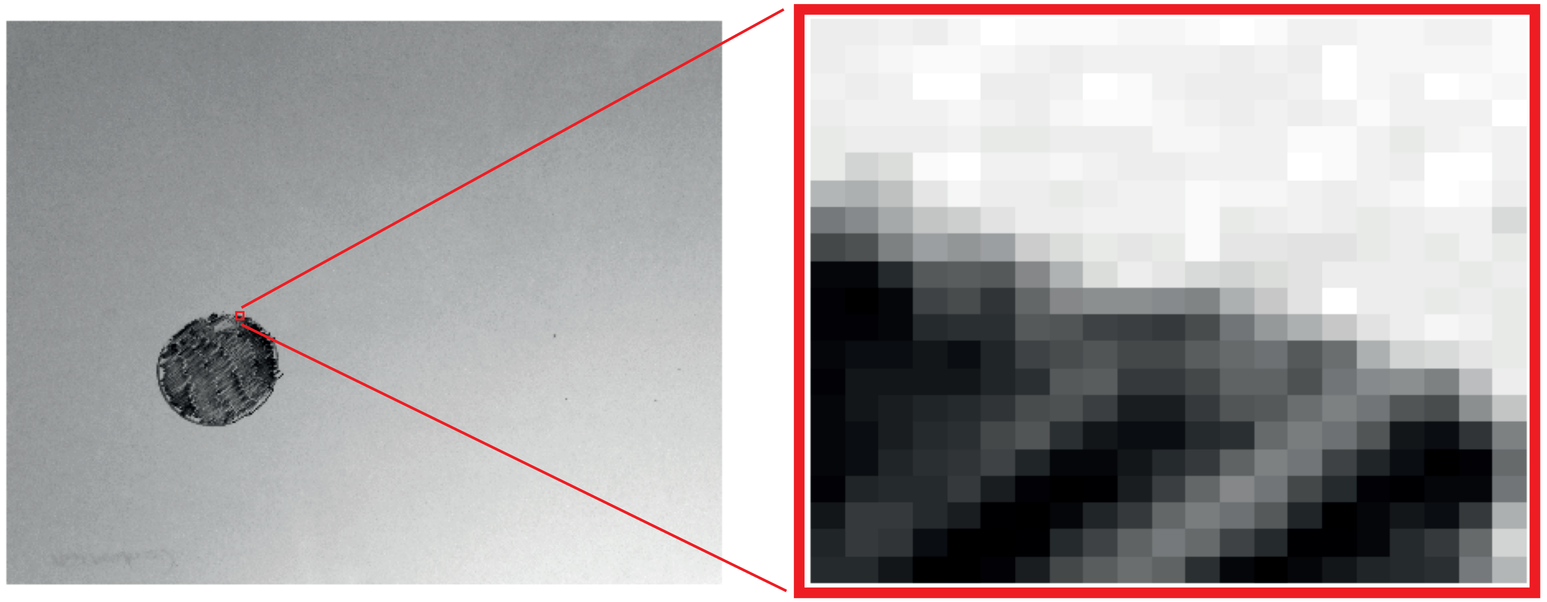
# Finding a Dot (getting quantitative information from a photograph)

Patch Kessler, 12.01.2012

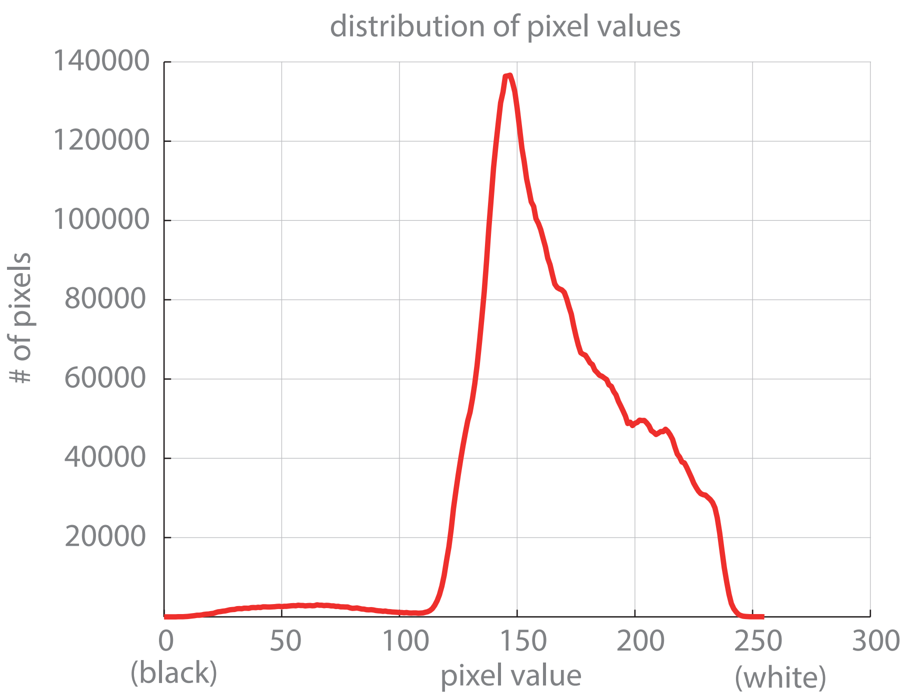
red channel of original image

zoomed in view

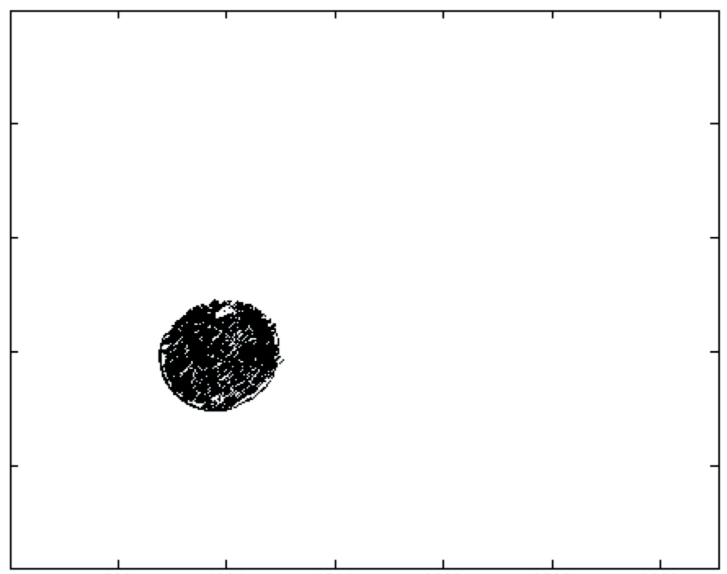


pixel values of zoomed in view

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 159 | 160 | 161 | 160 | 163 | 167 | 165 | 165 | 165 | 163 | 163 | 166 | 169 | 165 | 162 | 161 | 164 | 164 | 162 | 162 | 165 |
| 158 | 162 | 164 | 165 | 166 | 164 | 162 | 160 | 162 | 162 | 161 | 162 | 160 | 161 | 159 | 167 | 163 | 163 | 163 | 164 | 165 |
| 162 | 160 | 161 | 168 | 168 | 158 | 158 | 162 | 167 | 165 | 161 | 160 | 160 | 160 | 162 | 168 | 163 | 162 | 167 | 166 | 163 |
| 158 | 162 | 161 | 161 | 163 | 161 | 160 | 164 | 166 | 166 | 164 | 161 | 159 | 161 | 158 | 162 | 165 | 162 | 162 | 162 | 167 |
| 157 | 159 | 160 | 159 | 161 | 157 | 157 | 160 | 159 | 159 | 164 | 163 | 161 | 162 | 166 | 166 | 162 | 156 | 161 | 163 | 162 |
| 156 | 144 | 150 | 166 | 168 | 162 | 162 | 163 | 161 | 158 | 160 | 162 | 161 | 162 | 168 | 166 | 162 | 158 | 167 | 168 | 162 |
| 126 | 121 | 133 | 155 | 164 | 159 | 158 | 156 | 160 | 161 | 162 | 166 | 164 | 163 | 162 | 160 | 162 | 164 | 167 | 166 | 160 |
| 89 | 99 | 117 | 135 | 142 | 152 | 160 | 159 | 162 | 161 | 161 | 165 | 156 | 160 | 162 | 159 | 156 | 159 | 162 | 162 | 149 |
| 62 | 67 | 92 | 112 | 106 | 112 | 140 | 151 | 156 | 154 | 156 | 166 | 155 | 154 | 152 | 153 | 156 | 160 | 156 | 161 | 157 |
| 39 | 37 | 48 | 71 | 73 | 70 | 97 | 124 | 150 | 158 | 155 | 149 | 143 | 150 | 152 | 158 | 158 | 158 | 158 | 156 | 159 |
| 35 | 31 | 39 | 61 | 65 | 53 | 78 | 98 | 102 | 102 | 100 | 96 | 121 | 138 | 152 | 167 | 158 | 159 | 156 | 159 | 156 |
| 36 | 35 | 37 | 50 | 52 | 52 | 71 | 69 | 61 | 57 | 56 | 64 | 86 | 109 | 122 | 137 | 153 | 160 | 163 | 161 | 157 |
| 37 | 40 | 40 | 42 | 40 | 47 | 60 | 64 | 69 | 62 | 63 | 74 | 80 | 85 | 72 | 81 | 121 | 143 | 149 | 154 | 157 |
| 35 | 42 | 43 | 43 | 42 | 46 | 52 | 72 | 74 | 56 | 56 | 63 | 76 | 76 | 67 | 86 | 99 | 101 | 93 | 131 | 158 |
| 37 | 42 | 44 | 46 | 48 | 54 | 64 | 67 | 58 | 44 | 45 | 55 | 68 | 72 | 81 | 94 | 86 | 69 | 64 | 102 | 135 |
| 39 | 40 | 44 | 48 | 51 | 62 | 68 | 51 | 43 | 40 | 40 | 48 | 52 | 79 | 91 | 84 | 69 | 43 | 40 | 53 | 92 |
| 39 | 40 | 47 | 52 | 54 | 61 | 47 | 37 | 39 | 43 | 49 | 56 | 76 | 93 | 86 | 61 | 47 | 40 | 32 | 35 | 80 |
| 35 | 46 | 50 | 50 | 55 | 45 | 36 | 28 | 34 | 45 | 57 | 77 | 97 | 87 | 63 | 48 | 36 | 33 | 38 | 37 | 86 |
| 42 | 55 | 56 | 48 | 44 | 34 | 29 | 38 | 46 | 55 | 77 | 90 | 85 | 70 | 47 | 32 | 37 | 42 | 41 | 56 | 122 |
| 47 | 56 | 53 | 40 | 30 | 28 | 33 | 49 | 59 | 75 | 89 | 79 | 61 | 49 | 34 | 34 | 37 | 48 | 55 | 79 | 144 |
| 49 | 50 | 42 | 32 | 29 | 36 | 45 | 62 | 73 | 82 | 76 | 51 | 38 | 28 | 38 | 43 | 47 | 50 | 52 | 89 | 127 |

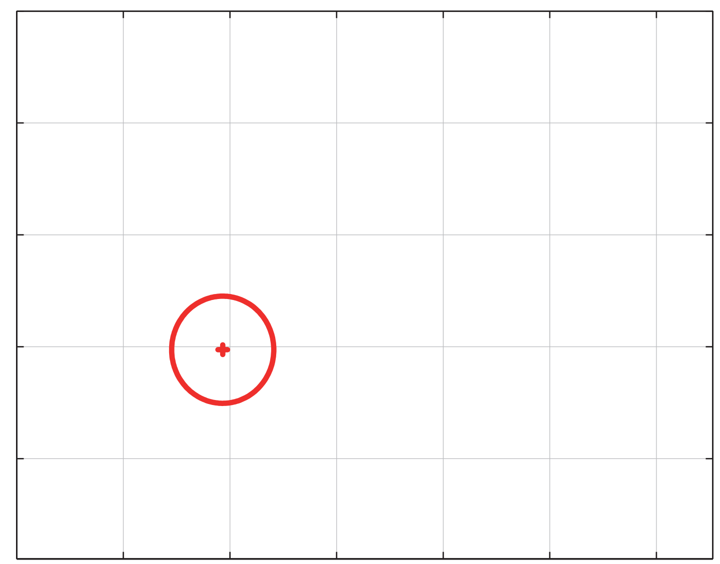



The distribution has two humps. The small one to the left of 100 corresponds to the black dot. The much larger one to the right corresponds to the white background.



Create a binary image B, the same size as the original image R, but with pixel values either 1 or 0.

B(i,j) = 1 if R(i,j) < 100,
B(i,j) = 0 otherwise.



Compute the average of the row coordinates of the pixels in B that equal 1. This is the row coordinate of the center of the dot. Likewise in the column direction.

Get a radius approximation from area = pi *radius^2. (The dot area is the number of pixels in B that equal 1.)

# Applying the SVD to an image (image compression)

Patch Kessler, 12.01.2012

## The singular value decomposition (SVD)

Any n x n matrix X has two associated sets of orthonormal vectors, $\{u_1, u_2, \ldots, u_n\}$ and $\{v_1, v_2, \ldots, v_n\}$, such that each set is a basis of $\mathbb{R}^n$, and such that

$$X = \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix} \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_n \end{bmatrix} \begin{bmatrix} v_1^T \\ v_2^T \\ \vdots \\ v_n^T \end{bmatrix} \quad \text{where } \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0.$$

This can be written as

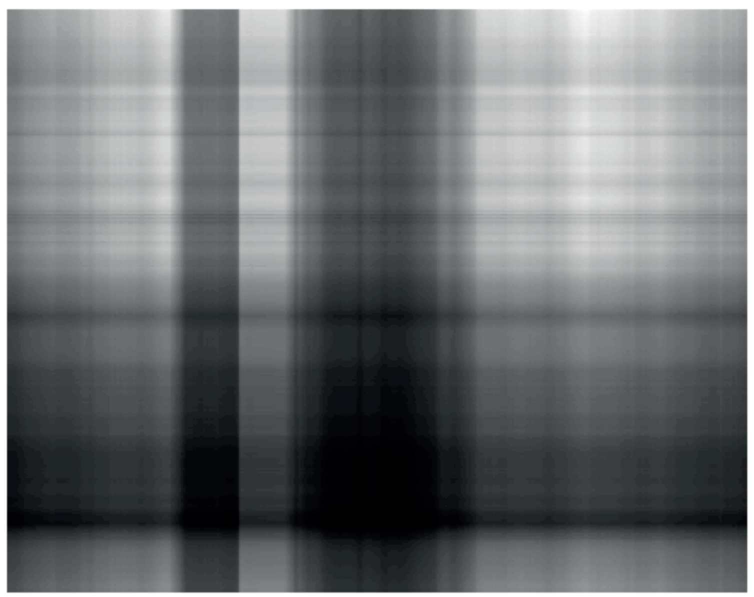
$$X = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \cdots + \sigma_n u_n v_n^T$$

Each of the $u_i v_i^T$ terms is called an "outer product" of the vectors $u_i$ and $v_i$. The sum of the first k of these from the SVD is as close to X as it is possible to get by summing k outer products.

## The SVD applied to an image

Suppose X is a picture of your friend sitting at a cafe. The sum of the first k terms of the SVD of X are shown below for k = 1, k = 10, k = 50, and k = 100.

The first term.



0.000817

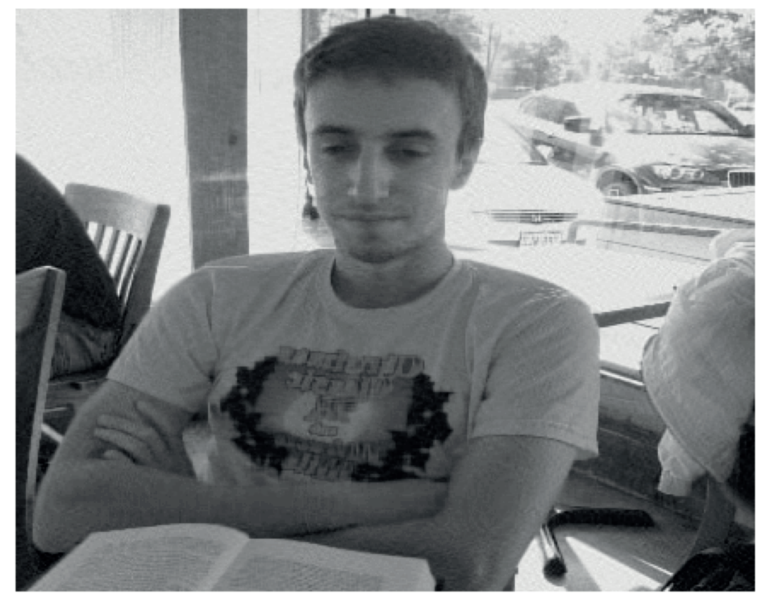
Sum of the first 10 terms.

0.008172

Sum of the first 50 terms.

0.040858

Sum of the first 100 terms.



0.081716

## compression

Note that X consists of n*n numbers, while the first k terms of the SVD of X consist of k*(2*n+1) numbers. When k is small, k*(2*n+1) is much less than n*n. Under each image above, we give the ratio k*(2*n+1)/(n*n). The image on the right, (which looks pretty good) only takes up 8% of the space of the original image!