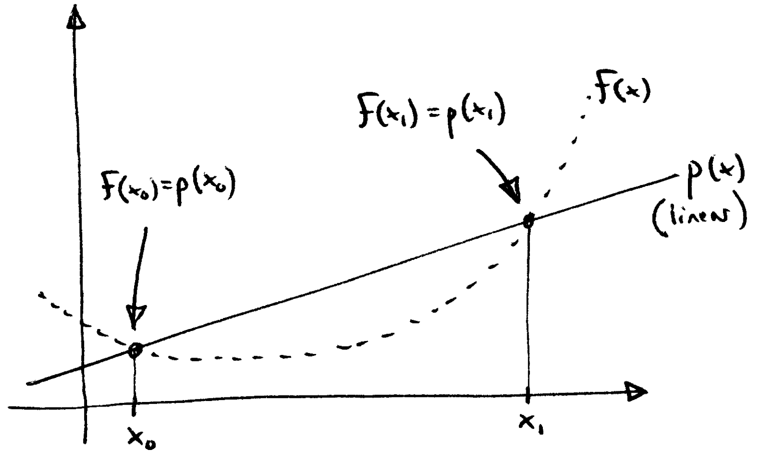
Summary of Polynomial Interpolation Error Formulas,

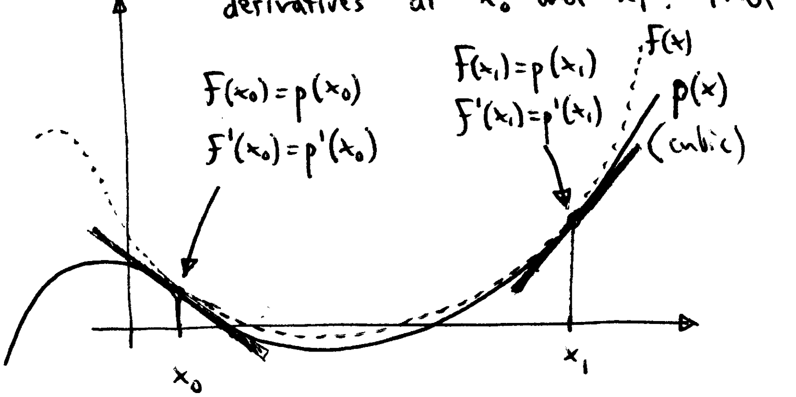
where $n=1$, and agreement between the polynomial and the given function occurs at the $n+1=2$ points x_0 and x_1.

Case 1: Let $p(x)$ be the degree <2 polynomial that agrees with a function $f(x)$ at x_0 and x_1 as illustrated. Then for every $\tilde{x}$, there is some ξ on the open interval $(\min(x_0,x_1,\tilde{x}), \max(x_0,x_1,\tilde{x}))$ such that



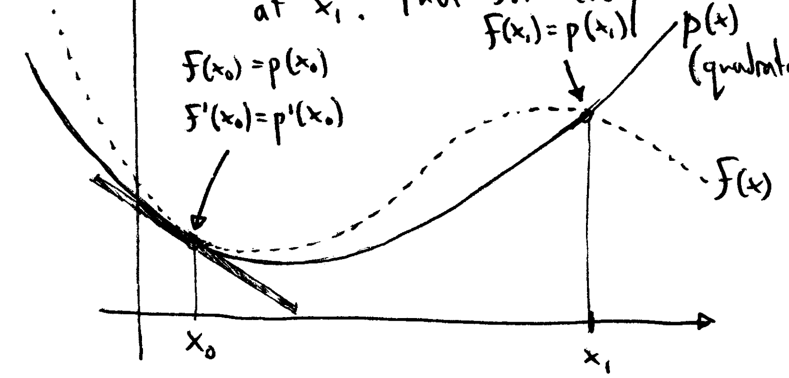
$$p(\tilde{x})-f(\tilde{x})=\frac{f^{[2]}(\xi)}{2!}(\tilde{x}-x_0)(\tilde{x}-x_1)$$

Case 2: Let $p(x)$ be the degree <4 polynomial that agrees with $f(x)$ and its first derivatives at x_0 and x_1. Then for every $\tilde{x}$, there is some ξ on the open interval $(\min(x_0,x_1,\tilde{x}), \max(x_0,x_1,\tilde{x}))$ such that



$$p(\tilde{x})-f(\tilde{x})=\frac{f^{[4]}(\xi)}{4!}(\tilde{x}-x_0)^2(\tilde{x}-x_1)^2$$

Case 3: Let $p(x)$ be the degree <3 polynomial that agrees with $f(x)$ and its first derivative at x_0, and that agrees with $f(x)$ (but not necessarily its first derivative) at x_1. Then for every $\tilde{x}$, there is some ξ on $(\min(x_0,x_1,\tilde{x}), \max(x_0,x_1,\tilde{x}))$ such that



$$p(\tilde{x})-f(\tilde{x})=\frac{f^{[3]}(\xi)}{3!}(\tilde{x}-x_0)^2(\tilde{x}-x_1)$$