Summary of Polynomial Interpolation Error Formulas

where \( n = 1 \), and agreement between the polynomial and the given function occurs at the \( n+1 = 2 \) points \( x_0 \) and \( x_1 \).

**Case 1**: Let \( p(x) \) be the degree < 2 polynomial that agrees with a function \( f(x) \) at \( x_0 \) and \( x_1 \), as illustrated. Then for every \( \bar{x} \), there is some \( \xi \) on the open interval \( (\min(x_0, x), \max(x_0, x)) \) such that

\[
p(\bar{x}) - f(\bar{x}) = \frac{f''(\xi)}{2!} (\bar{x} - x_0)(\bar{x} - x_1)
\]

**Case 2**: Let \( p(x) \) be the degree < 4 polynomial that agrees with \( f(x) \) and its first derivatives at \( x_0 \) and \( x_1 \). Then for every \( \bar{x} \), there is some \( \xi \) on the open interval \( (\min(x_0, x), \max(x_0, x)) \) such that

\[
p(\bar{x}) - f(\bar{x}) = \frac{f^{(4)}(\xi)}{4!} (\bar{x} - x_0)^2(\bar{x} - x_1)^2
\]

**Case 3**: Let \( p(x) \) be the degree < 3 polynomial that agrees with \( f(x) \) and its first derivative at \( x_0 \) and that agrees with \( f(x) \) (but not necessarily its first derivative) at \( x_1 \). Then for every \( \bar{x} \), there is some \( \xi \) on \( (\min(x_0, x), \max(x_0, x)) \) such that

\[
p(\bar{x}) - f(\bar{x}) = \frac{f''(\xi)}{2!} (\bar{x} - x_0)^2(\bar{x} - x_1)
\]