


Midterm 2 Ma 128a Prof 

1. Let $f(x)$ be analytic in an interval containing $[0, h]$ and let M be an upper bound on $|f^{(4)}(x)|$ in $[0, h]$. Let $T(x)$ be the 3rd order Taylor polynomial of $f(x)$ about $x=0$, and $P(x)$ be the Hermite polynomial that interpolates $f(x)$ and $f'(x)$ at $x=0$ and $x=h$.

a) Find an upper bound C on $|T(x) - f(x)|$ in $[0, h]$ in terms of M, h .

b) Find an upper bound on $|P(x) - f(x)|$ in $[0, h]$ in terms of M, h , which is less than C .

2. Let $y(x)$ be the natural cubic spline which interpolates the 3 points $(0,0)$, $(1,1)$, $(2,0)$.
 What are $y'(0)$ and $y''(1)$?

3. a) Let V be the subspace of \mathbb{R}^4 spanned by $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.
 What is $\min_{x \in V} |x - e_2|$, where $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$?

b) Let W be the subspace of \mathbb{R}^4 spanned by $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$. What is $\min_{x \in W} |x - e_2|$?

c) Find a basis of the subspace W^\perp of all vectors orthogonal to any x in W .

4. $T(x)$ is the temperature field in a bar:

$0 \leq x \leq \pi$, subject to temperature zero

at ends, $T(0) = T(\pi) = 0$, and heat source

$f(x)$ per unit length in $[0, \pi]$. Physically

realized $T(x)$ minimizes

$$E \equiv \int_0^{\pi} \left\{ \frac{1}{2} (T'(x))^2 - f(x)T(x) \right\} dx. \quad (1)$$

We seek approximation $T(x) \approx a \sin x$, by

substituting $a \sin x$ for $T(x)$ in (1) and

determining the value of the constant a

which minimizes $E(a)$.

a) Find this value of a , expressed as an integral.

Evaluate for $f(x) \equiv 1$ in $[0, \pi]$, and give the

approximate value of $T(\frac{\pi}{2})$ in this case.

b) For $f(x) \equiv 1$, the exact $T(x)$ has $T''(x) \equiv -1$.

Compute the exact value of $T\left(\frac{\pi}{2}\right)$.

(Hint: It behooves you to find quadratic $g(x)$ with $g''(x) \equiv -1$, and $g(0) = g(\pi) = 0$.)